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Stock prices and bond yields

Can their comovements be explained in terms of present value models?*

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Real stock prices do not show the relation to long-term interest rates that a simple rational expectations present value model would imply. Real stock prices drop when long-term interest rates rise (and rise when they fall) more than would be implied by this vector autoregression model. In contrast, over the last century changes in real stock prices have shown little correlation with changes in inflation rates, and according to the present value model they should show little correlation. These conclusions were reached from an analysis of annual data in the United States, 1871–1989, and the United Kingdom, 1918–1989.

1. Introduction

What should the relation be between changes in stock prices and changes in long-term bond yields? Is the observed relation right in the context of rational expectations present value models that base discount factors on market interest rates? There has long been confusion about the answers to these questions.

One argument has been that there should be a simple negative relation. By present value models an increase in expected future discount rates should, other things being equal, cause both stock prices to fall and long-term interest rates to

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rise; a fall in expected discount rates should have the opposite effect on both. Putting the same point in simpler terms, an increase in expected long-term bond yields would seem to make long-term bonds a more attractive investment, and so stock prices would have to fall to induce people to hold stocks.

That argument *might* be right, if certain implicit assumptions about stochastic properties of relevant variables are valid, but need not be. The problem with the argument is that the dividend stream that is discounted for stocks is radically different than the stream of coupons that is discounted for bonds. The implied differences in their stochastic properties can be relevant for the problem of the relations between the two assets from at least two perspectives.

First, the dividend stream on stocks is relatively stable in real terms, the dividend stream on long-term bonds in nominal terms. If there is substantial inflation, then these two streams can be dramatically different in ways that are correlated with the (nominal) discount rates. Therefore, if changes in nominal long-term bond yields reflect primarily inflationary expectations, then these changes should perhaps have little effect on stock prices.

Second, movements in long-term interest rates might be related to information about the future dividend stream on stocks. Consider the example of October 19, 1987, the day of the biggest one-day stock market crash in history. U.S. government bond prices did not fall, but actually rose; that is, long-term interest rates fell, and some interpreted these drops in interest rates as the result of adverse information about the outlook for corporate profits. This kind of positive relation between stock prices and long-term interest rates might possibly have a rational expectations interpretation: changes in long-term interest rates might carry information about changes in future dividends, and this information effect may offset the tendency for a negative relation between stock prices and bond yields.

There is no way to resolve these conflicting tendencies and answer the questions that introduced this paper except by seeing what information stock prices and bond yields carry about the future values of the *fundamentals* that enter into the present value relation.[1] This is done in the paper by means of vector autoregressive forecasting models for dividends and interest rates, based on the linearized version of the present value relationship proposed in Campbell and Shiller (1988a, b) in their study of the behavior of prices of corporate stocks, which they call dividend-ratio model. In this paper we shall (contrary to the

[1] Fama and French (1989) have also recently written on whether the observed relation between the bond and the stock market appears to make sense from a rational expectations viewpoint. They show that expected returns on stocks and long-term bonds are correlated with each other, a point already made by Campbell (1987), and that the expected returns are associated with variables related to 'longer-term aspects of business conditions'. However, they never formally use a present value model to test whether these longer-term aspects of business conditions are of the right magnitude from a rational expectations viewpoint.

practice in the work of Campbell and Shiller) also use, for consistency, this same dividend-ratio model to study the term structure of interest rates.

Our present value model here is of the simplest kind that disregards changes in risk premia: future dividends on stocks and coupons on bonds are discounted by the future short rates plus a constant risk premium. Barsky (1989) has stressed the possibility that the relation between stock prices and bond yields should be understood primarily in terms of a changing risk premia, but he did not do an econometric analysis of this model. We leave investigation of such possibilities to further work; the model considered here is recommended by its simplicity, and is in any event a step towards considering more general models.

As to the econometric representation of the time series considered in the empirical work, we will follow Campbell and Shiller (1988a, b), and estimate a VAR which is used (a) to test the restrictions imposed on the VAR by the present value relations and (b) to estimate what the dividend–price ratio on stocks and bonds should be if prices were set according to fundamentals, i.e., to estimate the 'theoretical' or 'warranted' dividend–price ratios.[2] This will allow us to compute covariances and correlations between theoretical real stock price changes and theoretical long-term interest rate changes, in order to verify whether there is any difference between these and the covariances and correlations between actual real stock price changes and actual long-term interest rate changes. This will also allow us to verify whether other measures of comovement between stock and bond markets are in accord with our theory. We will also compare actual and warranted relations between prices and yields with the change in inflation rates; some of these results may be interpreted as constituting some new evidence that bears on the 'overreaction to inflation' model introduced by Modigliani and Cohn (1979), as well as testing whether some stylized facts about asset pricing (e.g., the negative relationship between inflation and stock returns) can be explained by present value theories.

The paper is organized as follows: section 2 presents the theoretical present value relations, while section 3 introduces the econometric representation which we use to give empirical content to the theory. Section 4 derives the restrictions which the theory puts on the model, and shows what the metrics are that will be used to compare the two markets. Section 5 presents the results, section 6 discusses some small sample issues, while section 7 concludes the paper.

2. The theoretical models

The dividend-ratio model, the theoretical basis of the results in this paper, as advanced by Campbell and Shiller (1988a), is a model describing the behavior of

[2]In a companion paper [Beltratti and Shiller (1990)] we expand on the theory of estimation of warranted covariances and correlations, both under the null hypothesis represented by eq. (4) here and without this hypothesis. In the latter case, the point estimates of the warranted covariances and correlations are replaced by interval estimates.

the log dividend–price ratio δ_{jt} for asset j at time t. This model, a linearization of a present value model, may be called the dynamic Gordon model, after the model Myron Gordon proposed that made the dividend–price ratio in a steady state growth path equal the discount rate minus the growth rate of dividends. The Gordon model was derived under the assumption that dividend growth rates and discount rates are constant through time, and hence cannot be applied directly to time series data on dividend–price ratios, dividend growth rates, and discount rates. When these variables change through time, the dividend–price ratio on any date need not equal the instantaneous discount rate minus the growth rate of dividends on that date, since the dividend–price ratio depends as well on future discount rates and dividend growth rates. The dividend-ratio model instead makes the dividend–price ratio depend on the current and expected future one-period discount rates and one-period dividend growth rates, and gives relatively more weight to discount rates and dividend growth rates in the nearer future. When the dividend-ratio model is applied to long-term bonds (ideally consols), then, since the growth rate of dividends (coupons) is zero, the dividend-ratio model becomes the expectations theory of the term structure, relating the long-term yield to expected future one-period discount rates.

The theoretical concepts that underlie our analysis are very simple. Taking the dividend-ratio model and a vector autoregressive representation for the variables we wish to consider, we will infer what the log dividend–price ratio should be at each point of time, by substituting forecasted values of short-term discount rates and growth rates of dividends into the dividend-ratio model. We can then compare the covariance or correlation of this warranted log dividend–price ratio to other variables in the vector autoregression with the covariance or correlation of the actual log dividend–price ratio with these variables.

The log dividend–price ratio δ_{jt} is represented here as $d_{jt-1} - p_{jt}$, where d_{jt-1} is the log of the total dividends paid on asset j the preceding period (our theory assumes paid at the *end* of the preceding period, although dividends are in fact paid over the year) and p_{jt} is the log of the price of asset j at the beginning of year t. (A table of symbols for this paper appears in appendix A.) The theory says that

$$\delta_{jt} = \mathrm{E}_t \delta_{jt}^* \quad \text{where} \quad \delta_{jt}^* = \sum_{n=0}^{\infty} \rho_j^n (i_{jt+n} - \Delta d_{jt+n}) - \frac{k_j}{1-\rho_j}. \tag{1}$$

The operator E_t is the expectations operator conditional on all information at time t, the parameter ρ_j is a constant discount factor, i_{jt} is the one-period (continuously compounded) interest rate quoted at the beginning of time period t plus a constant risk premium for asset j. (In most respects, the constant risk premium will not affect our analysis with demeaned data.) Δd_{jt} is the log of total dividends paid on asset j in period t minus the log of total dividends paid in period $t-1$, and $k_j = -\rho_j \log(\rho_j) - (1-\rho_j)\log(1-\rho_j)$. The parameter ρ_j is determined by the point around which the linearization was made (it is only in our choice of ρ_j that the constant risk premium enters our analysis). When interest rates can be predicted to be high, then by eq. (1) the log dividend–price ratio will, other things equal, be high in this model. In this sense, the dividend ratio is a sort of long-term interest rate. When dividend growth is expected to be high, the log dividend–price ratio will, other things equal, tend to be low in this model. In this sense the dividend-ratio model asserts that dividend–price ratios are forecasters of future dividends.

Associated with the dividend-ratio model is an expression for an approximation ξ_{jt} to the log one-period gross return $h_{jt} = \log(\exp(\delta_{jt} - \delta_{jt+1}) + \exp(\delta_{jt})) + \Delta d_{jt}$:

$$\xi_{jt} = \delta_{jt} - \rho_j \delta_{jt+1} + \Delta d_{jt} + k_j. \tag{2}$$

In interpreting this expression, note that the $+\Delta d_{jt}$ (the change in log dividend between $t-1$ and t) serves to eliminate the lagged dividend from the expression, so that the linearized return is a function only of the beginning price, end price, and intervening dividend. The model (1) implies that the expectation conditional on all public information at time t of the excess return $\zeta_{jt} = \xi_{jt} - i_{jt}$ is unforecastable: $\mathrm{E}_t \zeta_{jt} = 0$. Moreover, with a terminal condition, the converse is also true: $\mathrm{E}_t \zeta_{jt} = 0$ implies (1).

The linearization allows us to use the linear theory of time series to study the present value models. A cost of linearizing is that we in effect disregard changes in the rate of discount for future interest rate changes and future log dividend changes. The importance of the errors introduced by the linearization were carefully analyzed, using several different metrics, in Campbell and Shiller (1988a). They showed evaluations of: 1) the accuracy of ξ_{jt} as an approximation of h_{jt}, 2) the accuracy of δ_{jt} as an approximation to the right-hand side of the second expression in model (1) where discount rates are equal to actual holding period yields, and using a terminal condition, 3) the accuracy of the right-hand side of the second expression in (1) as an approximation to an exact log dividend–price ratio computed under a lognormality assumption, and 4) comparisons of regressions of linearized returns ξ_{jt} on information with regressions of actual returns h_{jt} on information.

Model (1) is so formulated that it is easy to use either real (inflation corrected) or nominal data to test the present value relation. Using nominal data has the advantage of allowing us to ignore inflation in the sense that the empirical work is performed in terms of the difference between the nominal interest rate and the rate of growth of nominal dividends. This is the road we pursue in our analysis of the bond market. For the stock market however we consider a real version of the model, which in the empirical work separately considers the real interest rate and the rate of growth of real dividends,[3] and therefore may be used, as we will show more clearly below, to compare the two assets by using a variety of different metrics.

After having motivated this important difference in our treatment of the two markets, we come to a description of the variables that will be used in the rest of the paper. For our stock market data $j = s$ (s for stocks); according to the real version of the model we can interpret $i_{st} - \Delta d_{st}$ in (1) as the difference between the real interest rate and the rate of growth of real dividends. Here, i_{st} is the nominal continuously compounded short-term interest rate (plus a constant risk premium) minus the inflation rate. In principle we would like to have data for the nominal interest rate quoted at the beginning of year t on a one-year bill plus a risk premium for stocks; in fact, our data are not exactly this: we use here with U.S. data the one-year return for rolling over prime commercial paper rate, investing for six months in January and again in July.

Since the January price index is used to deflate both dividends paid last year and stock prices at the beginning of this year, there is no effect of the use of real variables on the variable δ_{st}, which can however be interpreted as the difference between the log of real dividends d_{st} paid over the year on the portfolio of stocks comprising a stock index, minus the log of p_{st} that is the log of the real stock price at the beginning of the year.

With this set of variables the return described by (2), denoted by ξ_{st}, is to be interpreted as a one-period real gross return for stocks. The parameter ρ_s is taken to be $\exp(g - r_s)$, where g is the average growth rate of real dividends and r_s is the average real return on stocks over our sample.

For bonds (taken in our theory to be consols, as with our U.K. data), eq. (1) could be called an expectations theory of the term structure. The variable d_{bt} ($j = b$ for bonds) is set at the log of the nominal coupon, which is constant through time. The variable p_{bt} is the natural log of the nominal price of the bond at the beginning of the year, and hence δ_{bt} is the log of the consol yield at the beginning of year t. The variable i_{bt} is the same nominal interest rate used above for calculating the real interest rate, plus a possibly different constant risk premium; again the constant risk premium will not affect our data which are demeaned. With this set of variables the return described by (2), denoted by ξ_{bt}, is to be interpreted as a one-period nominal gross return for bonds. Here, ρ_b is $\exp(-r_b)$, where r_b is the average value of nominal rates of return on long bonds over the sample.[4] We omit to use real dividend data for bonds because the

[3] The nominal and the real interpretation of the model can be considered different ways of making the model suitable to empirical testing by turning nonstationary variables into stationary variables.

[4] Discounting is done with a nominal rate for bonds because nominal coupons are fixed; it is done with a real rate for stocks because real dividends are more nearly following a constant growth path than are nominal dividends. Note also that the risk premia of stocks versus bonds affect the constant discount factors used in (1).

change in the log real coupon might not be stationary, due to nonstationary inflation rates, which will be seen inconvenient for the econometric representation described below.

The expectations theory of the term structure appears here in a rather unusual form, since the log of the consol yield is set to a distributed lead of expected future short rates where the sum of the coefficients is not one but $1/(1-\rho_b)$, or approximately one divided by the discount rate. If we linearize $\ln(R)$ around $\bar{R}$, then $\ln(R)$ is given approximately equal to $\bar{R} + (R - \bar{R})/\bar{R}$; this explains why the sum of the coefficients is not one.[5]

3. Econometric representation

In order to study the theoretical correlation between the two markets, we need to use an econometric representation that jointly considers the variables describing the two assets. We stack into a vector x_t the following variables:

$$x_t = [\delta_{st}, \Delta d_{st-1}, i_{st-1}, S_{bt}, \Delta i_{bt-1}]' . \tag{3}$$

The time subscripts on the various elements of x are chosen so that all information in x_t is known at the beginning of the year t. Most elements of this vector have been defined before (or see appendix A; data sources are in appendix B). The first three elements of x refer to stocks: δ_{st} is the dividend–price ratio for stocks, Δd_{st-1} is the rate of growth of real dividends, and i_{st-1} is the real short-term interest rate at the beginning of time $t-1$ (ideally, on an instrument that matures at the end of time $t-1$). The fourth element S_{bt} equals $(1-\rho_b)\delta_{bt} - i_{bt-1}$, which is the difference between $(1-\rho_b)$ times the log of the consol yield for January of period t minus the continuously compounded nominal interest rate for year $t-1$. S_{bt} is a sort of 'spread' between the long rate and the short rate, or slope of the term structure; the coefficient $(1-\rho_b)$ enters for the reason indicated in the last paragraph of the preceding section.[6] The fifth element Δi_{bt-1} is the change in the average continuously compounded short-term rate from $t-2$ to $t-1$.

Eq. (1) applied to the first three elements of (3) is the dividend-ratio model for stock prices. Eq. (1) applied to the last two elements of (3) is just a version of the expectations theory of the term structure of interest rates, according to which

[5] We compared the linearization proposed in this paper with the one used by Shiller (1979) by means of various metrics, for example computing the correlation between actual and approximated returns. We found that in general the log-linearized version of this paper works slightly better than the Shiller (1979) version.

[6] The reason why we consider the spread and not the level of the long-term interest rate is related to stationarity, as explained later on in the section.

the spread between long-term and short-term bonds reflects expectations of future changes in short-term interest rates.[7]

This representation allows us to analyze the correlation between stock prices and long-term interest rates, as we shall see below, because here we observe separately both the rate of growth of real dividends and the interest rate, and this is necessary to isolate interest rates from dividends in the dividend–price ratio expression. It will also allow us to look directly at the change in real stock prices, rather than just at the excess returns, as will be discussed below. Moreover, the system contains the (lagged) change in the inflation rate, given by $\Delta\pi_{t-1} = \Delta i_{bt-1} - i_{st-1} + i_{st-2}$, so that we can study the correlation of changes in stock prices with changes in inflation rates.

The form of x_t in (3) is dictated by our presumptions as to the probable stationarity of the various variables. We suppose that the log dividend–price ratio δ_{st} for stocks is a stationary stochastic process; indeed previous studies have rejected a unit root model for this series [see Campbell and Shiller (1988a)]. (We report below nonetheless a Monte Carlo experiment that tests for the effects of such a unit root in the dividend yield for our econometric results.) Our model implies that δ_{st} will be stationary if growth rates of real dividends and real interest rates are stationary stochastic processes. However, we believe that the log dividend–price ratio δ_{bt} for bonds may not be stationary, or at least show strong low-frequency components. Since the nominal dividends on bonds are fixed in nominal, rather than real, terms, nonstationarity of inflation rates translates into nonstationarity of δ_{bt}. That is why δ_{bt} does not appear in our model by itself, but rather in the form of the spread variable S_{bt}. Our model implies that S_{bt} is the market's expectations of the future changes in interest rates, and will be therefore be stationary so long as the *changes* in short-term interest rates are stationary.

Another aspect of the form of the vector x_t is dictated by our concern that our results not be affected by the possibility that economic agents have superior information, beyond that represented in our vector x_t. Surely they do use more information variables than we can readily incorporate into our models, e.g., qualitative information about the stance of monetary policy. But we have included in our vector x_t a variable representing the left-hand side of our model (1) itself. For stocks, this is δ_{st}, for bonds it is S_{bt}. Thus, a variable summarizing, in effect, all *relevant* information is included in the vector x_t.

Even though under the theory (1) all relevant information is contained in δ_{st} and S_t, for the purpose of *testing* the theory other information variables included in x_t can be important [see Beltratti (1989)]. Indeed the vector x_t described earlier can be considered as a mix of fundamental and information

[7] In order to see that (1) implies that the spread reflects expectations of future changes in interest rates, subtract the past level of nominal interest rates from both sides of (1) and rearrange.

variables: when testing the restrictions for the stock market the last two variables in (3) are just information variables, and when the restrictions on bonds are tested, the first three variables are information variables. In this way the fundamental variables for one market naturally represent information variables for the other market.

4. The restrictions imposed by the present value formulas

Following Campbell and Shiller (1988a, b) we assume an autoregressive process for the vector x:

$$(I - A(L))x_t = a_t, \tag{4}$$

where $A(L)$ is a matrix polynomial in the lag operator L, $A(0) = 0$, and a_t is white noise, with a covariance matrix which can have nonzero contemporaneous correlations. We estimate vector autoregressions of order 1 to 3. For testing purposes we do not need to determine the 'right' number of lags, since under the null hypothesis all the information should be incorporated in the current asset price.

The restrictions that the present value models considered in this paper put on a first-order vector autoregression [when x contains the elements described in eq. (3)] are:[8]

$$\delta_{st} = \delta'_{st} \quad \text{where} \quad \delta'_{st} = (e3 - e2)'(Ax_t + \rho_s A^2 x_t + \cdots)$$

$$= (e3 - e2)' A(I - \rho_s A)^{-1} x_t, \tag{5}$$

for the stock market, and

$$S_{bt} = S'_{bt} \quad \text{where} \quad S'_{bt} = e5' A(I - \rho_b A)^{-1} x_t, \tag{6}$$

for the bond market. We shall refer to δ'_{st} and S'_{bt} as the theoretical log dividend–price ratios and spreads, respectively, since these are the values predicted by our theory (1) with the time series model. Here, e_i is a vector whose elements are zero except for the ith element, which is one. Eqs. (5) and (6) imply

[8] For vector autoregressions of order larger than 1 we can just rewrite the system in a first-order companion form. The restrictions in (5), (6), and (7) are valid only for a first-order vector autoregression, but can easily be reformulated for higher-order VARs. See Campbell and Shiller (1988a, b).

the following restrictions on the estimated coefficients:

$$e1'(I - \rho_s A) + (e2 - e3)'A = 0, \tag{7a}$$

$$e4'(I - \rho_b A) - e5'A = 0. \tag{7b}$$

In the empirical section we test the restrictions for the two markets using Wald tests. But the basic purpose of this paper is to find what certain correlations *should* be given the present value model and compare these with actual correlations. As suggested by Campbell and Shiller (1988b) we can use (5) and (6) to calculate the theoretical asset values which would prevail if the present value model were true, and use them to compute various measures of theoretical correlation between the two markets.

We can compute theoretical excess returns on stocks and bonds, the excess returns that would obtain if economic agents were forecasting using the VAR model; these are:

$$\zeta'_{st} = \xi'_{st} - i_{st} = \delta'_{st} - \rho_s \delta'_{st+1} + \Delta d_{st} - i_{st}, \tag{8a}$$

$$\zeta'_{bt} = \xi'_{bt} - i_{bt} = (1 - \rho_b)^{-1}(S'_{bt} - \rho_b S'_{bt+1} - \Delta i_{bt}). \tag{8b}$$

To fulfill the basic mission of this paper, to examine whether the relation between the stock and bonds markets is in accordance with the theory (1), we will first compare the correlation between actual excess returns in the stock market and actual excess returns in the bond market with the correlation between their theoretical counterparts given in (8).

Due to the separation between real interest rates and the rate of growth of real dividends, we can also calculate the change in stock prices from the elements of x_t given in (3) as

$$\Delta p_{st} = \Delta d_{st-1} - \Delta \delta_{st}, \tag{9}$$

where p_{st} is stock price in real terms.[9] Similarly, we can compute from (4) the change in log bond yields by

$$\Delta \delta_{bt} = (1 - \rho_b)^{-1}(S_{bt} - S_{bt-1} + \Delta i_{bt-1}). \tag{10}$$

[9] The change in the real price would not be recoverable from the information in a system including only the difference between the rate of growth of nominal dividends and the nominal interest rate.

If the efficient markets model (1) is true and if the series are generated by the vector autoregression specified, then, whether or not there is other information in the vector used to forecast, these, by (5) and (6), should equal:

$$\Delta p'_{st} = \Delta d_{st-1} - \Delta\delta'_{st}, \tag{11}$$

$$\Delta\delta'_{bt} = (1-\rho_b)^{-1}(S'_{bt} - S'_{bt-1} + \Delta i_{bt-1}). \tag{12}$$

We shall compare the correlation of Δp_{st} with $\Delta\delta_b$ with the correlation of $\Delta p'_{st}$ with $\Delta\delta'_{bt}$ to see if the correlations between price changes and changes in long-term interest rates are what they should be if the efficient markets hypothesis (1) were true. Moreover, the correlation of each of these with the change in the one-year inflation rate ($\Delta\pi_t = \Delta i_{bt} - \Delta i_{st}$) will enable us to tell if markets respond appropriately to inflation. Note that we cannot look either at the correlation between the change in the nominal stock price and any other variable, or at the correlation of any variable with the rate of inflation because we have assumed that inflation must be differenced to induce stationarity.

5. Results

Table 1 presents results for the United States with the full sample 1871–1989, table 2 for the United States with a postwar sample 1948–1989, table 3 for the United Kingdom with the full sample 1918–1989, and table 4 for the United Kingdom with a postwar sample 1948–1989.

The results show that there is a negative correlation between the change in actual real log stock prices Δp_s and the change in actual long-term interest rates $\Delta\delta_b$ (panel A of each table); it is close to -0.4 for the U.S. and close to -0.6 in the U.K. in both the full and postwar samples. There should not, according to the estimates based on the present value model, be such a strong negative correlation: $\text{corr}(\Delta p'_{st}, \Delta\delta'_{bt})$ is much closer to zero in both the U.S. and the U.K. regardless of sample or lag length in the vector autoregression.

The results also show that there is a positive correlation between the actual excess return ζ_s in the stock market and the actual excess return ζ_b in the bond market (panel B of each table); it is close to 0.4 for the U.S. and close to 0.6 in the U.K. in both the full and postwar samples. There should not, according to the estimates based on the present value model, be such a strong positive correlation: $\text{corr}(\zeta_s, \zeta_b)$ is much closer to zero both in the U.S. and the U.K. regardless of sample or lag length in the vector autoregression. These panel B results might be regarded as essentially a duplication of the panel A results, since excess returns used in panel B are highly correlated with the changes used in panel A; however, the panel B results do not make use of any price deflator. The price

Table 1

United States, 1871–1989.[a]

A. Relations of log real price change Δp_s to long-term interest rate change $\Delta \delta_b$

A.1. Actual relations: $\text{corr}(\Delta p_s, \Delta \delta_b)$ -0.427, $\text{cov}(\Delta p_s, \Delta \delta_b)$ -0.006449

A.2. Theoretical relations:

Lags	$\text{corr}(\Delta p_s', \Delta \delta_b')$	$\text{cov}(\Delta p_s', \Delta \delta_b')$
1	−0.156	0.001678
	(0.154)	(0.001612)
2	−0.226	−0.002862
	(0.212)	(0.002742)
3	−0.084	−0.000971
	(0.232)	(0.002643)

B. Relations of stocks excess returns ζ_s and bonds excess returns ζ_b

B.1. Actual relations: $\text{corr}(\zeta_s, \zeta_b)$ 0.395, $\text{cov}(\zeta_s, \zeta_b)$ 0.005974

B.2. Theoretical relations:

Lags	$\text{corr}(\zeta_s', \zeta_b')$	$\text{cov}(\zeta_s', \zeta_b')$
1	0.037	0.000410
	(0.127)	(0.001399)
2	0.097	0.001225
	(0.203)	(0.002580)
3	−0.061	−0.000668
	(0.214)	(0.002384)

C. Relations with the change in inflation rate $\Delta\pi$

C.1. Actual relations:

$\text{corr}(\Delta p_s, \Delta\pi)$	−0.031	$\text{cov}(\Delta p_s, \Delta\pi)$	−0.000530
$\text{corr}(\Delta \delta_b, \Delta\pi)$	−0.028	$\text{cov}(\Delta \delta_b, \Delta\pi)$	−0.000229
$\text{corr}(\zeta_s, \Delta\pi)$	0.251	$\text{cov}(\zeta_s, \Delta\pi)$	0.004229
$\text{corr}(\zeta_b, \Delta\pi)$	0.067	$\text{cov}(\zeta_b, \Delta\pi)$	0.000557
$\text{corr}(\xi_s, \Delta\pi)$	−0.048	$\text{cov}(\xi_s, \Delta\pi)$	−0.000789

C.2. Theoretical relations:

Lags	$\text{corr}(\Delta p_s', \Delta\pi)$	$\text{corr}(\Delta \delta_b', \Delta\pi)$	$\text{corr}(\zeta_s', \Delta\pi)$	$\text{corr}(\zeta_b', \Delta\pi)$	$\text{corr}(\xi_s', \Delta\pi)$
1	−0.032	0.299	0.361	−0.250	−0.053
	(0.162)	(0.148)	(0.144)	(0.148)	(0.161)
2	−0.067	0.285	0.311	−0.248	−0.086
	(0.166)	(0.160)	(0.147)	(0.160)	(0.165)
3	−0.121	0.237	0.347	−0.205	−0.143
	(0.163)	(0.163)	(0.152)	(0.167)	(0.161)

Table 1 (continued)

Lags	$\mathrm{cov}(\Delta p'_s, \Delta\pi)$	$\mathrm{cov}(\Delta\delta'_b, \Delta\pi)$	$\mathrm{cov}(\zeta'_s, \Delta\pi)$	$\mathrm{cov}(\zeta'_b, \Delta\pi)$	$\mathrm{cov}(\xi'_s, \Delta\pi)$
1	− 0.000380	0.002521	0.004402	− 0.002108	− 0.000616
	(0.001919)	(0.001339)	(0.001859)	(0.001305)	(0.001859)
2	− 0.000841	0.002685	0.003972	− 0.002273	− 0.001057
	(0.002090)	(0.001509)	(0.002037)	(0.001478)	(0.002037)
3	− 0.001261	0.002464	0.003577	− 0.002052	− 0.001476
	(0.001753)	(0.001720)	(0.001717)	(0.001691)	(0.001717)

D. P-values for the Wald test of the model restrictions (7)

Lags	Stocks	Bonds
1	0.273	0.021
2	0.075	0.007
3	0.027	0.001

[a]Standard errors are in parentheses.

Table 2

United States, 1948–1989.[a]

A. Relations of log real price change Δp_s to long-term interest rate change $\Delta\delta_b$

A.1. Actual relations: $\mathrm{corr}(\Delta p_s, \Delta\delta_b)$ − 0.409, $\mathrm{cov}(\Delta p_s, \Delta\delta_b)$ − 0.007264

A.2. Theoretical relations:

Lags	$\mathrm{corr}(\Delta p'_s, \Delta\delta'_b)$	$\mathrm{cov}(\Delta p'_s, \Delta\delta'_b)$
1	− 0.277	− 0.002267
	(0.204)	(0.001736)
2	− 0.159	− 0.000941
	(0.331)	(0.001980)
3	− 0.025	− 0.000117
	(0.331)	(0.001522)

B. Relations of stocks excess returns ζ_s and bonds excess returns ζ_b

B.1. Actual relations: $\mathrm{corr}(\zeta_s, \zeta_b)$ 0.366, $\mathrm{cov}(\zeta_s, \zeta_b)$ 0.006085

B.2. Theoretical relations:

Lags	$\mathrm{corr}(\zeta'_s, \zeta'_b)$	$\mathrm{cov}(\zeta'_s, \zeta'_b)$
1	0.064	0.000364
	(0.180)	(0.001025)
2	0.106	0.000451
	(0.339)	(0.001462)
3	− 0.130	− 0.000337
	(0.368)	(0.000977)

Table 2 (continued)

C. Relations with the change in inflation rate $\Delta\pi$

C.1. Actual relations:

$\text{corr}(\Delta p_s, \Delta\pi)$	−0.263	$\text{cov}(\Delta p_s, \Delta\pi)$	−0.002530
$\text{corr}(\Delta\delta_b, \Delta\pi)$	0.135	$\text{cov}(\Delta\delta_b, \Delta\pi)$	0.000830
$\text{corr}(\zeta_s, \Delta\pi)$	−0.063	$\text{cov}(\zeta_s, \Delta\pi)$	−0.000558
$\text{corr}(\zeta_b, \Delta\pi)$	−0.092	$\text{cov}(\zeta_b, \Delta\pi)$	−0.000574
$\text{corr}(\xi_s, \Delta\pi)$	−0.265	$\text{cov}(\xi_s, \Delta\pi)$	−0.002509

C.2. Theoretical relations:

Lags	$\text{corr}(\Delta p_s', \Delta\pi)$	$\text{corr}(\Delta\delta_b', \Delta\pi)$	$\text{corr}(\zeta_s', \Delta\pi)$	$\text{corr}(\zeta_b', \Delta\pi)$	$\text{corr}(\xi_s', \Delta\pi)$
1	−0.505	0.389	−0.103	−0.353	−0.517
	(0.153)	(0.228)	(0.251)	(0.241)	(0.154)
2	−0.456	0.291	−0.233	−0.245	−0.462
	(0.139)	(0.225)	(0.211)	(0.249)	(0.143)
3	−0.640	0.319	−0.408	−0.224	−0.650
	(0.113)	(0.262)	(0.220)	(0.333)	(0.115)

Lags	$\text{cov}(\Delta p_s', \Delta\pi)$	$\text{cov}(\Delta\delta_b', \Delta\pi)$	$\text{cov}(\zeta_s', \Delta\pi)$	$\text{cov}(\zeta_b', \Delta\pi)$	$\text{cov}(\xi_s', \Delta\pi)$
1	−0.002332	0.002298	−0.000354	−0.001958	−0.002305
	(0.000892)	(0.001482)	(0.000867)	(0.001457)	(0.000867)
2	−0.001722	0.000994	−0.000704	−0.000744	−0.001687
	(0.000755)	(0.000783)	(0.000747)	(0.000799)	(0.000747)
3	−0.001802	0.000700	−0.000825	−0.000385	−0.001784
	(0.000618)	(0.000587)	(0.000608)	(0.000582)	(0.000608)

D. P-values for the Wald test of the model restrictions (7)

Lags	Stocks	Bonds
1	0.002	0.046
2	0.000	0.126
3	0.000	0.000

[a]Standard errors are in parentheses.

deflator is the series among those we use that is most vulnerable to measurement error.[10]

These findings might be described as finding that the stock market 'overreacts' to the bond market. But of course there is nothing in these results that

[10]We also analyzed these correlations and covariances by means of a four-variable VAR, including the difference between the nominal interest rate and the rate of growth of nominal dividends instead of the real interest rate and the rate of growth of real dividends. This VAR does not require use of the price deflator at all; market participants are not assumed to have information about real quantities. This is what we called in the text the nominal interpretation of the model for stock prices. The results are very similar to those reported in the various tables for the five-variable VAR.

indicates that the direction of causality is from the bond market to the stock market.

A question that naturally arises in evaluating this finding of 'overreaction' of stocks to bond yields is whether the excessive negative correlation between changes in stock prices and changes in long-term bond yields is best thought of as an excessively negative correlation with changes in long-term interest rate changes or of as an excessively negative correlation with changes in the inflation expectations component of long-term interest rates. Lacking data on the expectations of inflation, we cannot answer this directly, but we can use our model to compare the correlation of each of the two markets with the change in the inflation rate to its theoretical or warranted value (panel C). It is striking that with the full samples, both in the U.S. and in the U.K., there is no substantive correlation between changes in actual real stock prices and changes in inflation. Nor should there be, according to the present value model: the full sample $\mathrm{corr}(\Delta p'_{st}, \Delta\pi_t)$ is negligible in both countries. It is also striking that with the shorter postwar samples, both in the U.S. and in the U.K., there is a negative correlation between changes in real stock prices and changes in inflation. And there should be such a negative correlation, according to the present value model: the short sample $\mathrm{corr}(\Delta p'_{st}, \Delta\pi_t)$ is negative in both countries. Thus, the rational expectations model (1) does on the whole justify the observed relation of changes in real stock prices with changes in inflation.

The results with bonds and inflation might be summarized by saying that the correlation between changes in bond yields δ_b and changes in inflation $\Delta\pi$ is usually not very big in absolute value, and should not be very big. The only exception to this summary is that with postwar British data (table 4) there is some positive correlation between changes in bond yields and changes in inflation, while our analysis indicates that the correlation should be negative.

Table 3

United Kingdom, 1918–1989.[a]

A. Relations of log real price change Δp_s to long-term interest rate change $\Delta\delta_b$

A.1. Actual relations: $\mathrm{corr}(\Delta p_s, \Delta\delta_b)$ − 0.582, $\mathrm{cov}(\Delta p_s, \Delta\delta_b)$ − 0.016418

A.2. Theoretical relations:

Lags	$\mathrm{corr}(\Delta p'_s, \Delta\delta'_b)$	$\mathrm{cov}(\Delta p'_s, \Delta\delta'_b)$
1	0.084 (0.158)	0.000809 (0.001570)
2	0.092 (0.203)	0.000750 (0.001665)
3	0.007 (0.239)	0.000005 (0.001610)

Table 3 (continued)

B. Relations of stocks excess returns ζ_s and bonds excess returns ζ_b

B.1. Actual relations: $\mathrm{corr}(\zeta_s, \zeta_b)$ 0.608, $\mathrm{cov}(\zeta_s, \zeta_b)$ 0.015717

B.2. Theoretical relations:

Lags	$\mathrm{corr}(\zeta_s', \zeta_b')$	$\mathrm{cov}(\zeta_s', \zeta_b')$
1	0.006 (0.164)	0.000005 (0.001407)
2	0.065 (0.207)	0.000450 (0.001448)
3	0.158 (0.225)	0.000919 (0.001392)

C. Relations with the change in inflation rate $\Delta\pi$

C.1. Actual relations:

$\mathrm{corr}(\Delta p_s, \Delta\pi)$	−0.119	$\mathrm{cov}(\Delta p_s, \Delta\pi)$	−0.002385
$\mathrm{corr}(\Delta\delta_b, \Delta\pi)$	0.155	$\mathrm{cov}(\Delta\delta_b, \Delta\pi)$	0.001573
$\mathrm{corr}(\zeta_s, \Delta\pi)$	0.049	$\mathrm{cov}(\zeta_s, \Delta\pi)$	0.000899
$\mathrm{corr}(\zeta_b, \Delta\pi)$	−0.139	$\mathrm{cov}(\zeta_b, \Delta\pi)$	−0.001408
$\mathrm{corr}(\xi_s, \Delta\pi)$	−0.139	$\mathrm{cov}(\xi_s, \Delta\pi)$	−0.002674

C.2. Theoretical relations:

Lags	$\mathrm{corr}(\Delta p_s', \Delta\pi)$	$\mathrm{corr}(\Delta\delta_b', \Delta\pi)$	$\mathrm{corr}(\zeta_s', \Delta\pi)$	$\mathrm{corr}(\zeta_b', \Delta\pi)$	$\mathrm{corr}(\xi_s', \Delta\pi)$
1	−0.219 (0.157)	−0.008 (0.178)	0.230 (0.170)	0.033 (0.177)	−0.226 (0.154)
2	−0.067 (0.167)	−0.126 (0.181)	0.128 (0.188)	0.141 (0.189)	−0.073 (0.167)
3	−0.162 (0.204)	−0.135 (0.235)	0.144 (0.231)	0.145 (0.245)	−0.166 (0.202)

Lags	$\mathrm{cov}(\Delta p_s', \Delta\pi)$	$\mathrm{cov}(\Delta\delta_b', \Delta\pi)$	$\mathrm{cov}(\zeta_s', \Delta\pi)$	$\mathrm{cov}(\zeta_b', \Delta\pi)$	$\mathrm{cov}(\xi_s', \Delta\pi)$
1	−0.001811 (0.001367)	−0.000007 (0.001486)	0.001738 (0.001313)	0.000266 (0.001446)	−0.001835 (0.001313)
2	−0.000360 (0.000915)	−0.000689 (0.001025)	0.000611 (0.000903)	0.000734 (0.001016)	−0.000390 (0.000903)
3	−0.000769 (0.001008)	−0.000579 (0.001041)	0.000614 (0.000993)	0.000595 (0.001033)	−0.000785 (0.000993)

D. P-values for the Wald test of the model restrictions (7)

Lags	Stocks	Bonds
1	0.000	0.000
2	0.000	0.000
3	0.000	0.000

[a]Standard errors are in parentheses.

Table 4

United Kingdom, 1948–1989.[a]

A. Relations of log real price change Δp_s to long-term interest rate change $\Delta\delta_b$

A.1. Actual relations: $\mathrm{corr}(\Delta p_s, \Delta\delta_b)$ − 0.637, $\mathrm{cov}(\Delta p_s, \Delta\delta_b)$ − 0.023247

A.2. Theoretical relations:

Lags	$\mathrm{corr}(\Delta p'_s, \Delta\delta'_b)$	$\mathrm{cov}(\Delta p'_s, \Delta\delta'_b)$
1	− 0.208 (0.182)	− 0.001626 (0.001620)
2	− 0.310 (0.312)	− 0.002514 (0.003243)
3	− 0.072 (0.337)	− 0.000359 (0.001670)

B. Relations of stocks excess returns ζ_s and bonds excess returns ζ_b

B.1. Actual relations: $\mathrm{corr}(\zeta_s, \zeta_b)$ 0.662, $\mathrm{cov}(\zeta_s, \zeta_b)$ 0.023065

B.2. Theoretical relations:

Lags	$\mathrm{corr}(\zeta'_s, \zeta'_b)$	$\mathrm{cov}(\zeta'_s, \zeta'_b)$
1	0.228 (0.172)	0.001661 (0.001431)
2	0.458 (0.272)	0.003458 (0.003099)
3	0.253 (0.327)	0.001062 (0.001406)

C. Relations with the change in inflation rate $\Delta\pi$

C.1. Actual relations:

$\mathrm{corr}(\Delta p_s, \Delta\pi)$	− 0.201	$\mathrm{cov}(\Delta p_s, \Delta\pi)$	− 0.002027
$\mathrm{corr}(\Delta\delta_b, \Delta\pi)$	0.379	$\mathrm{cov}(\Delta\delta_b, \Delta\pi)$	0.001926
$\mathrm{corr}(\zeta_s, \Delta\pi)$	− 0.130	$\mathrm{cov}(\zeta_s, \Delta\pi)$	− 0.001265
$\mathrm{corr}(\zeta_b, \Delta\pi)$	− 0.370	$\mathrm{cov}(\zeta_b, \Delta\pi)$	− 0.001864
$\mathrm{corr}(\xi_s, \Delta\pi)$	− 0.203	$\mathrm{cov}(\xi_s, \Delta\pi)$	− 0.001968

C.2. Theoretical relations:

Lags	$\mathrm{corr}(\Delta p'_s, \Delta\pi)$	$\mathrm{corr}(\Delta\delta'_b, \Delta\pi)$	$\mathrm{corr}(\zeta'_s, \Delta\pi)$	$\mathrm{corr}(\zeta'_b, \Delta\pi)$	$\mathrm{corr}(\xi'_s, \Delta\pi)$
1	− 0.088 (0.239)	0.091 (0.241)	0.180 (0.231)	− 0.091 (0.243)	− 0.082 (0.229)
2	− 0.022 (0.275)	− 0.197 (0.253)	0.210 (0.262)	0.217 (0.263)	− 0.014 (0.275)
3	− 0.181 (0.194)	− 0.107 (0.214)	0.013 (0.229)	0.101 (0.230)	− 0.175 (0.192)

Table 4 (continued)

Lags	$\text{cov}(\Delta p_s', \Delta\pi)$	$\text{cov}(\Delta\delta_b', \Delta\pi)$	$\text{cov}(\zeta_s', \Delta\pi)$	$\text{cov}(\zeta_b', \Delta\pi)$	$\text{cov}(\xi_s', \Delta\pi)$
1	− 0.000241 (0.000660)	0.000366 (0.000971)	0.000478 (0.000627)	− 0.000349 (0.000935)	− 0.000225 (0.000627)
2	− 0.000007 (0.000902)	− 0.000704 (0.000984)	0.000680 (0.000881)	0.000731 (0.000969)	− 0.000004 (0.000881)
3	− 0.00440 (0.000510)	− 0.000275 (0.000579)	0.000003 (0.000503)	0.000243 (0.000574)	− 0.000428 (0.000503)

D. P-values for the Wald test of the model restrictions (7)

Lags	Stocks	Bonds
1	0.000	0.000
2	0.000	0.000
3	0.000	0.000

[a]Standard errors are in parentheses.

Panel D of the tables reports Wald tests of the restrictions (7) of the model. In both the U.S. and the U.K. case, the Wald test rejects the restrictions on unpredictability of one-period excess returns, both for bonds and stocks and for both periods. This agrees with previous tests for the stock market [see Campbell and Shiller (1988b)].

6. Small sample properties

In Campbell and Shiller (1989) the small sample properties of estimates and standard errors in a dividend-ratio model were studied. These studies were carried further in Beltratti (1989) and Beltratti and Shiller (1990). The results were interpreted as indicating some small sample bias for estimated parameters.

Here, we report some Monte Carlo experiments which are focussed on the sampling distributions of correlations and covariances, and which provide p-values for the Wald tests in table 1 above based on empirical distributions.

In the first Monte Carlo experiment, we first derived an estimate of the vector autoregression (4) in the first-order case subject to the restrictions (7a) and (7b). This was done by estimating a vector autoregression for the vector $\tilde{x}_t = Sx_t$, where S is the identity matrix where the second row is replaced by $[\rho_s \ -1 \ 1 \ 0 \ 0]$ and the last (fifth) row was replaced by $[0 \ 0 \ 0 \ \rho_b \ 1]$. Then $\tilde{x}_t = \tilde{A}\tilde{x}_t + \tilde{u}_t$, where $\tilde{A} = SAS^{-1}$ and $\tilde{u}_t = Su_t$. Restriction (7a) asserts that the second row of $\tilde{A}$ equals $[1 \ 0 \ 0 \ 0 \ 0]$ and restriction (7b) implies that the last row of $\tilde{A}$ equals $[0 \ 0 \ 0 \ 1 \ 0]$. Imposing these values, estimating the equations for the other three rows of $\tilde{A}$ with the data and sample of table 1, and then computing $A = S^{-1}\tilde{A}S$, we derive an estimated vector-autoregressive model for use in constructing data for the Monte Carlo experiments, and we performed 1000 iterations with the sample size of table 1, initializing at zero.

The average (across the 1000 iterations) correlation between $\Delta p_s'$ and $\Delta\delta_b'$ was 0.030, not much different from the mean correlation of Δp_s with $\Delta\delta_b$ (across the 1000 iterations) of 0.031; there is very little bias for this correlation. The Monte Carlo experiment also confirms the standard errors on these correlations reported in table 1. The standard deviation across iterations of the correlation between $\Delta p_s'$ and $\Delta\delta_b'$ was 0.155, a little higher than the asymptotic standard error of 0.154 reported in table 1, and the standard deviation of the covariance between Δp_s and $\Delta\delta_b$ was 0.095. These Monte Carlo results confirm that the difference between the actual correlation of -0.427 and the theoretical correlation of -0.156 reported in table 1 is significant: in only 27 of the 1000 iterations was the absolute difference between these correlations so great.

There was also only a slight bias in our estimate of the covariation between $\Delta p_s'$ and $\Delta\delta_b'$: the average (across the 1000 iterations) covariance was 0.00243, only slightly different from the average covariance between Δp_s and $\Delta\delta_b$ of 0.00267. But the absolute difference between the actual and theoretical covariances observed in table 1 was exceeded in our Monte Carlo experiment by the absolute difference between the actual and theoretical covariances in fully 450 of the 1000 iterations, confirming our impression that the differences between actual and theoretical covariances are not significant.

These Monte Carlo results were similarly confirming our conclusions regarding covariances and correlations between excess returns and between various variables and inflation rates. The empirical p-value for the Wald test is 0.280 for stocks (compared with 0.273 reported in table 1) and 0.032 for bonds (compared with 0.021 reported in table 1).

In a second Monte Carlo experiment we replaced the first row of $\tilde{A}$ with the first row of the identity matrix, thereby imposing that the log dividend–price ratio is a random walk and has a unit root, and proceeded with 1000 iterations as before. Again, the results were confirming of our general conclusions. In only 53 of the 1000 iterations did the absolute difference between the correlation between $\Delta p_s'$ and $\Delta\delta_b'$ and the correlation between Δp_s and $\Delta\delta_b$ exceed the value observed in table 1. The p-values for the Wald test given in table 1 were increased slightly: now they are 0.331 for stocks and 0.032 for bonds; still, they are not far from the p-values reported in table 1.

7. Conclusions

In the introduction to this paper we alluded to the simple theory that changes in real stock prices should be negatively correlated with changes in long-term interest rates, since the rate of discount has opposite effects on both. We conclude that, if we assume a simple present value model, then, in view of the nature of the variability of discount rates and dividends in relation to information available in advance of this variability, there should indeed be generally a slight negative correlation between changes in real stock prices and changes in long-term interest rates, but that the actual observed correlation is more negative in the U.S. and U.K. data than it should be.

We also found generally that excess returns in the stock market correlate too much with excess returns in the bond market when compared with what the correlation should be in the terms defined here. This result was found fairly consistently in both long and short samples, in both the U.S. and the U.K., and with varying VAR lag lengths.

We did not find that there was evidence of any excessive correlation of either the stock or bond markets with changes in inflation rates. It should be borne in mind, of course, that we are here talking of changes in actual one-year inflation rates and not of changes in the expectations of long-run inflation that are relevant to changes in long-term bond prices.

Appendix A: Table of symbols

A.1. Latin symbols

a_t = white noise.

$A(L)$ = matrices whose elements are polynomials in lag operator L, used in expression (4).

d_{jt} = log dividend paid on asset j at time t. The theory assumes that it is paid at the *end* of period t, just before p_{jt} is observed. It is real for stocks ($j = s$) and nominal for bonds ($j = b$).

h_{jt} = one-period return on asset j; real for stocks ($j = s$) and nominal for bonds ($j = b$).

i_{jt} = one-year discount rate for asset j at time t; real for stocks ($j = s$) and nominal for bonds ($j = b$); $i_{st} = i_{bt} - \pi_t +$ a constant risk premium differential.

j = subscript j indicates asset; when $j = s$ the asset is stocks, when $j = b$ the asset is bonds.

p_{jt} = log price of asset j at (the beginning of) time t; real for stocks ($j = s$) and nominal for bonds ($j = b$).

S_{bt} = long-short interest rate spread at time t; $S_{bt} = (1 - \rho_b)\delta_{bt} - i_{bt-1}$.

x_t = vector whose elements are known to public at time t, expression (3).

A.2. Greek symbols

δ_{jt} = log dividend–price ratio for asset j; $\delta_{jt} = d_{jt-1} - p_{jt}$.

Δ = first-difference operator; $\Delta x_t = x_t - x_{t-1}$.

ξ_{jt} = approximate return on asset j between time t and $t+1$, expression (2); real for stocks ($j = s$) and nominal for bonds ($j = b$).

ζ_{jt} = approximate excess return $= \xi_{jt} - i_{jt}$. Since it is an excess return, it is both 'real' and 'nominal' regardless of j.

π_t = inflation rate between t and $t+1$.

ρ_j = discount factor used in the linearization (1) for asset j. It is determined by the point chosen for linearization. In the case of stocks ($j = s$) we took ρ_s to be $\exp(g - r_s)$, where g is the mean growth rate of real dividends and r_s is the mean real return over the full sample. For bonds ($j = b$) $\rho_b = \exp(-r_b)$; the growth of dividends is zero and r_b is the mean nominal return over the full sample.

Note: A prime (′) added to a scalar variable denotes 'theoretical' or 'warranted' value, which is the value it should have if theory (1) is true. See expressions (5), (6), (8a), (8b), (11), and (12).

Appendix B: Sources of data

The U.S. stock price data used here are the same (except for updating) as in Shiller (1989b, ch. 26) and the U.K. stock price data are the same (except for updating) as in Shiller (1989a). The interest rate data, both for the U.S. and the U.K., are the same (except for updating) as in Shiller (1989b, ch. 13). The above references give further information about data sources.

For the United Kingdom, the stock price index P_{st} (used to calculate p_{st}) 1919–1986 is the Barclay's de Zoete Wedd (BZW) Equity Index for the end of year $t-1$, and the dividend series D_{st} (used to calculate d_{st}) is the associated dividends for the index for all of year t. The price deflator 1929 = 1.00 used to convert nominal to real quantities is from Friedman and Schwartz (1982, table 4.9, col. 4, pp. 132–134) and updated. The BZW index was also used by Bulkley and Tonks (1988) in their study of the efficiency of the U.K. stock market. The U.K. nominal short-term interest rate i_{bt} is the three-month prime bank bill rate, averaged over the year. The U.K. nominal long-term interest rate series $\exp(\delta_{bt})$ is the British consol yield at the beginning of year t.

For the United States, P_{st}, the annual stock price index 1871–1988, is the January Standard and Poor Composite stock price index, and the dividend series D_{st} is the corresponding dividends (total for year). The Standard and Poor Composite Stock Price Index and corresponding dividends per share adjusted to index, starting 1926, are from Standard and Poor Statistical Service. Before 1926, the dividends per share are adapted from Cowles (1939); see Shiller (1989). The price deflator used to convert nominal to real quantities is the January producer price index from the Bureau of Labor Statistics; see Shiller (1989). The producer price index starting in 1913 is the January all commodities producer price index from the U.S. Bureau of Labor Statistics. For years before 1913, it is linked to the January index of all commodities prices from Warren and Pearson (1935, pp. 13–14). The short rate is the annual return on 4- to 6-month prime commercial paper, computed from January and July figures under the assumption of a six-month maturity. The long rate 1871 to 1936 is the January unadjusted railroad bond yield from Macaulay (1938), after that it is the Moody AAA bond yield average for January.

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